Dronacharya Group of Institutions, Gr. Noida

Department of Applied Sciences (First Year)

Even Semester (2020-2021)

Objective Question Bank

Subject Name & Code: Engineering Mathematics II (KAS 203T)

Unit-IV (COMPLEX VARIABLE-DIFFERNTIATION)

1. Cauchy-Riemann equations in Cartesian form is/ are

   (a) $u_x = v_y, u_y = -v_x$ (b) $u_x = v_y, u_y = v_x$ (c) $u_{xx} + v_{yy} = 0$ (d) None of these.

2. Cauchy-Riemann equations in polar form is/ are

   (a) $u_r = \frac{1}{r} v_\theta, \frac{1}{r} u_\theta = -v_r$ (b) $u_x = v_y, u_y = v_x$ (c) $u_{rr} + v_\theta = 0$ (d) None of these.

3. A function which is analytic everywhere in finite complex plane is known as.

   (a) Entire function (b) Holomorphic function (c) meromorphic function (d) None

4. Let $f(z) = u + iv$ be a complex valued function. Where $v = 3xy^2$, then

   a) f is analytic for any choice of u

   b) f is analytic for suitable choice of u

   c) f is analytic only when $u =$ constant

   d) f can't be analytic for any choice of u.

5. If a function $f(z)$ is continuous at z_0, then

   a) $f(z)$ is differentiable at z_0

   b) $f(z)$ is not necessarily differentiable at z_0

   c) $f(z)$ is analytic at z_0.

   d) None of the above.

6. The only function among the following that is analytic, is

   a) $f(z) = \mathrm{Re}(z)$ b) $f(z) = \mathrm{Im}(z)$ c) $f(z) = \bar{z}$ d) $f(z) = \sin z$

7. If $w = u(x, y) + iv(x, y)$ is an analytic function of $z = x + iy$, then $\frac{dw}{dz}$ equals

   a) $\frac{\partial w}{\partial x}$ b) $-i\frac{\partial w}{\partial x}$ c) $i\frac{\partial w}{\partial y}$ d) $-i\frac{\partial w}{\partial y}$

8. If $f(z) = u(x, y) + i\, v(x, y)$ is analytic, then $f'(z)$ equals

a) $\frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$ b) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial x}$ c) $\frac{\partial u}{\partial y} - i\frac{\partial v}{\partial x}$ d) $i\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$

9. u, v are called conjugate harmonic function if

a) u, v are harmonic function and $u + i\,v$ may not be analytic function.

b) u, v are harmonic function

c) $u + i\,v$ is an analytic function

d) u, v are harmonic function and $u + i\,v$ is an analytic function.

10. An analytic function is

a) Infinitely differentiable

b) not necessarily differentiable

c) finitely differentiable.

d) None of these.

11. Let $u(x, y) = 2x(1 - y)$, for all real x and y.
Then a function $v(x, y)$ so that $f(z) = u(x, y) + i\,v(x, y)$ is analytic, is

a) $x^2 - (y - 1)^2$

b) $(x - 1)^2 - y^2$

c) $(x - 1)^2 + y^2$

d) $x^2 - (y - 1)^2 + C$

12. At $z = 0$, the function $f(z) = z^2\bar{z}$

a) is analytic

b) differentiable

c) doesn't satisfy CR equation

d) Satisfy CR equations but not differentiable

13. Which of the following cannot be the real part of an analytic function.

a) $x^2 - y^2$ b) $x^2 + y^2$ c) $\cos x \cosh y$ d) $\frac{1}{2}\log\left(x^2 + y^2\right)$

14. The harmonic conjugate of $u(x, y) = (x - 1)^3 - 3xy^2 + 3y^2$ is

a) $3y\left(1 + x^2\right) - y^3$

b) $3x\left(1 + y^2\right)$

c) $(y - 1)^{-3} + 3xy^2 - 3y^2$

d) $(x - 1)^3 i + 3xy^2 i - 3y^2 i$

14. A function *f* (*z*)is analytic function if

(a) Real Part of f(z) is analytic

(b) Imaginary part of f(z) is analytic

(c) Both real and imaginary part of $f(z)$ is analytic

(d) none of the above

15. If u and v are harmonic functions then $f(z) = u + iv$ is

a) Analytic function

(b) need not be analytic function

(c) Analytic function only at $z = 0$

(d)none of the above

16. If $f(z)=x+ay+i(bx+cy)$ is analytic the value of $a,b,\&c$ are
(A) $c=1,a=-b$ (B) $a=1,c=-b$ (C) $b=1,a=-c$ (D) $a=1=b=c$

17. A point at which a function ceases to be analytic is called a
a. Singular point (b)Non-Singular point (c) Regular point (d) Non-regular point

18. A function v is called a conjugate harmonic function for a harmonic function u in Ω whenever
(a) $f=u+iv$ is analytic (b) u is analytic (c) v is analytic(d) $f=u-iv$ is analytic

19. If $f(z)=x^3+ax^2y+bxy^2+cy^3$ is analytic the value of $a,b,\&c$ are
(A) $a=3i,b=-3,c=-i$ (B) $a=3i,b=3,c=-i$
(C) $a=3i,b=-3,c=i$ (D) $a=-3i,b=-3,c=-i$

21. There exist no analytic functions f such that
*a)*Re $f(z)=y-2x$ (b) Re $f(z)=y^2-2x$ (c) Re $f(z)=y^2-x^2$ (d) Re $f(z)=y-x$

22. If $e^{ax}\cos y$ is harmonic, then value of a is
(a)i (b) 0 (c) -1 (d)2

23 The harmonic conjugate of $2x-x^3+3xy^2$ is
(a) $x-3x^2y+y^3$ (b) $2y-3x^2y+y^3$ (c) $y+3x^2y+y^3$ (d) $2y+3x^2y-y^3$

24 The function $e^x(\cos y-i\sin y)$ is
(a) analytic (b) not analytic (c) analytic when z=0 (d) analytic when z=i

25. If $f(z)$ is analytic then $\overline{f}(\overline{z})$ is
(a)analytic (b)not analytic (c) analytic when z=0 (d) analytic when z=1

26. Points at which $f(z)=\dfrac{z^2-1}{z^2-3z+2}$ is not analytic are
(a) 1&-1 (b) i & -i (c) 1& i (d) 1 & 2

27 . The points at which $f(z)=\dfrac{1}{1+z^2}$ is not analytic
(a) 1&-1 (b) i & -i (c) 1& i (d) -1& -i

28. The points coincide with their transformations are known as
(a) fixed points (b) critical points (c) singular points (d) None of these

29. A translation of the type $w = \alpha z + \beta$ where α and β are complex constants, is known as
(a)translation (b)magnification (c) linear transformation (d) bilinear transformation

30. A mapping that preserves angles between oriented curves both in magnitude and in sense is called a/an.....mapping.
(a) informal (b)signal (c)conformal (d)formal

31. The mapping defined by an analytic function $f(z)$ is conformal at all points *z* except at points where
(a)$f'(z) = 0$ (b) $f'(z) \neq 0$ (c) $f'(z) > 0$ (d) $f'(z) < 0$

32. The invariant points of the transformation $w = \dfrac{z}{2-z}$ are
(a) -1,1 (b) 0,-1 (c) 0,1 (d)-1,1

33. The fixed points of the transformation $w = \dfrac{z-1}{z+1}$ are
(a) -1,1 (b) i&-i (c) 0,-1 (d)0,1

35. The mapping $w = z + z^{-1}$ transforms circles of constant radius into
(a) confocal ellipses (b)hyperbolas (c)circles (d)parabolas

36. The bilinear transformation that maps the points $0, i, \infty$ respectively into $0, 1, \infty$ is *w*
(a) $\dfrac{1}{z}$ (b) $-z$ (c) iz (d) $-iz$

37. The invariant points of the transformation $w = \dfrac{1+z}{1-z}$ are
a) i, i b) $i, \quad -i$ c) $1- \ i, \quad 1+i$ d) $-i, \quad -1+i$

38. By the transformation $w = ze^{\frac{\pi i}{4}}$, the line $x = 0$ is transformed into the line
a) $v = -u$ b) $v = u$ c) $u + v = 1$ d) $v = 0$

39. Critical points of $w = \dfrac{\alpha z + \beta}{\gamma z + \delta}, \alpha\delta - \beta\gamma \neq 0$ are
a) $-\dfrac{\delta}{\gamma}$ b) $-\dfrac{\delta}{\gamma}$ and ∞ c) $-\dfrac{\delta}{\gamma}$ and 0 d) 0 and ∞

40. The mapping $w = z^2 - 2z - 3$ is
a) conformal within $|z| = 1$
b) not conformal at z=1
c) not conformal at $z = -1$ and $z = 3$
d) everywhere conformal

41. Under the mapping $w = z + 2 - i$, the image of live $y = 0$ is,

a) $\text{Im}(w) = 1$ b) $\text{Im}(w) = -1$ c) $\text{Re}(w) = 1$ d) $\text{Re}(w) = -1$

42. The bilinear teams formation w which maps the point $0, 1, \infty$ is the z- plane onto the points- $i, \infty, 1$ is $w-$ plane is

a) $\frac{z-1}{z+i}$ b) $\frac{z-i}{z+1}$ c) $\frac{z+i}{z-1}$ d) $\frac{z+1}{z-i}$

43. The bilinear transformation whose fixed points are 1 and 2 is w =..........

(a) w= (z + 2)/(4+z)(b) w= (z + 2)/(4-z)(c) w= (z - 2)/(4-z)(d) none of these

44. The fixed points of the transformation $w = z^3$ are

(a) 0,1 (b)0,-1 (c) -1,1 (d) none of these

45. The points of invariant of the transformation w = (2z + 3) / (z + 2) is.....

(a)$z = \pm(3)^{1/2}$ (b) z = ±(3) (c)$z = \pm(3i)^{1/2}$ (d)z = ±(3i)

46. If f(z) is an analytic function and $v = y^2 - x^2$,then conjugate harmonic function is

(a) 2xy - c (b)$2x^{2y}$ + c (c)2xy + c (d)$2(y^2 - x^2) + c$

47. $e^{2x}(x\cos 2y - y\sin 2y)$ is

(a) not analytic (b)analytic (c) analytic when z =2i (d)analytic when z =i

48. If real part of an analytic function $f(z)$ is $x^2 - y^2 - y$ then it's imaginary part is

(A) $2xy + c$ (B) $x^2 + 2xy + c$ (C) $2xy - y + c$ (D) $2xy + x + c$

49. If imaginary part of an analytic function $f(z)$ is $2xy + y$ then it's imaginary part is

(A) $x^2 + y^2 - y$ (B) $x^2 - y^2 - x$ (C) $x^2 - y^2 + x$ (D) $x^2 - y^2 + y$

50. Harmonic conjugate of $u(x, y) = e^y \cos x$ is

(A) $e^x \cos y + c$ (B) $e^x \sin y + c$ (C) $e^y \sin x + c$ (D) $-e^y \sin x + c$

51. The function f(z)= $\sqrt{|xy|}$ is not analytic at the origin but C -R conditions are satisfied

(A) At origin only (B) Everywhere (C) Both A &B (D) None of these.

52. The function f(z)=$\frac{x^2 y^5 (x+iy)}{x^4 + y^{10}}, z \neq 0$ and f(0)=0 is not analytic

(A) At origin only (B) Everywhere (C) Both A &B (D) None of these.

53. The analytic function whose real part is $x^3-3xy^2+3x^2-3y^2+1$.

(A) z^3+3z^2+1+ic. (B) z^3+3z^2+5+ic. (C) z^3+z^2+1+ic. (D) None of these.

54. Find the regular function whose imaginary part is $e^{-x}(x\cos y+y\sin y)$, $f(0)=1$ is

(A) $1 + ze^{-z}$ (B) z-12 (C) z+7 (D) None of these.

55. If f(z) is an analytic function of z, then $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f'(z)|$

(A) 0 (B) 5 (C) 7 (D) 12.

56. Find the constants a, b and c such that $f(z) = -x^2 + xy + y^2 + i(ax^2 + bxy + y^2)$ is analytic.

(A) $a = -1/2, b = -2, c = 1/2$ (B) $a = -1/2, b = -2, c = 3/2$ (C) $a = -1/2, b = -3, c = 1/2$ (D) None of these.

57. If $u = x^2 - y^2$ then u is

(A) Harmonic (B) Analytic (C) Harmonic Conjugate (D) Conformal